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String Model in Differential Forms

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The string model is formulated in terms of two-dimensional differential forms of arbitrary rank. The local supersymmetric string action with local conformal and Lorentz symmetries is constructed. The connection with topological quantum field theory is discussed. Covariant quantization of the model is investigated. The critical space-time dimension is found to be $d=4$.

1. INTRODUCTION

Recent hope of unifying the fundamental interactions is connected with string theory. In the low-energy limit this theory gives a good description of gravity and the gauge fields. The cancellation condition of gauge anomalies determines the gauge group to be either $SO(32)$ or $E_8 \otimes E_8$ (Green *et al.*, 1987). On the other hand, one determines the dimension of the space-time in which the string is embedded from a condition of cancellation of the conformal anomalies (Polyakov, 1981). For known models this dimension is equal to $d=10$ or $d=26$. Hence, compactification of the additional dimensions is necessary. But the mechanism of such a compactification is too arbitrary (Green *et al.*, 1987) for the theory to be fundamental. Thus, the problem of constructing a string model directly in four-dimensional space-time has been attracting growing attention (Ellis, 1987; Kawai *et al.*, 1987; Antoniadis and Bachas, 1988).

The standard string models are formulated in terms of two-dimensional scalar and spinor fields (Polyakov, 1981). We develop here the string model on the basis of our program (Solodukhin, 1988, 1989) of using only two-dimensional differential forms of arbitrary rank. It is known that antisymmetric tensor fields (or differential forms) play an important role in various

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aspects of string theory (Rohm and Witten, 1986; Dominique, 1986; Teitelboim, 1986; Rabin, 1986). On the other hand, the connection of external forms with topology makes such a consideration especially attractive. We exploit extensively the possibility of describing fermions by differential forms using an equation first suggested by Ivanenko and Landau (1928) and widely discussed in the literature (Kàhler, 1962; Graf, 1978; Becher and Joos, 1983; Benn and Tucker, 1983; Ivanenko and Obukhov, 1985; Ivanenko *et al.*, 1985; Bullinaria, 1986).

2. BOSONIC SECTOR OF THE MODEL

In standard string models (Green *et al.*, 1987; Polyakov, 1981; Brink *et al.*, 1976; Deser and Zumino, 1976) the string is described by the action

$$S=\frac{1}{2}\int d^2z\,\sqrt{g}\,g^{\mu\nu}\,\partial_\mu X^i\,\partial_\nu X^i \tag{1}$$

where $g_{\mu\nu}$ is the metric on the world-sheet of a moving string, and scalar functions $X^i(z)$, $i=1,\ldots,d$, realize the embedding of this world-sheet into the d-dimensional space-time.

It is easy to see that the action (1) might be transformed into

$$S=\frac{1}{2}\int *\,dX^i\wedge dX^i \tag{1'}$$

A natural generalization of this expression is to substitute zero-forms X^i by the full inhomogeneous exterior forms on the world-sheet:

$$\phi^i=\varphi^i+\varphi^i_\mu\,dz^\mu+\tfrac{1}{2}\varphi^i_{\mu\nu}\,dz^\mu\wedge dz^\nu$$

Thus, one can suggest the following generalization of (1′):

$$S=\frac{1}{2}\int *\,(d-\delta)\phi^i\wedge(d-\delta)\phi^i \tag{2}$$

where d and δ are external differential and codifferential operators, respectively (Dubrovin *et al.*, 1979).

In the string theory conformal symmetry plays a special role (Green *et al.*, 1987). It is easy to see that (2) is not conformal invariant in general. However, it is invariant under global transformations:

$$\begin{aligned} g_{\mu\nu}&\to e^{2\sigma}g_{\mu\nu}, &\qquad \varphi_\mu&\to e^{\sigma}\varphi_\mu\\ \varphi&\to\varphi, &\qquad \varphi_{\mu\nu}&\to e^{2\sigma}\varphi_{\mu\nu}\end{aligned} \tag{3}$$

Localizing these, we find that under conformal transformation

$$(d-\delta)\phi \to [(d-\delta)\phi\,]_{\text{glob}} + e^{-\sigma}\nabla^{\mu}\sigma\varphi_{\mu} + e^{\sigma}\partial_{\mu}\sigma\varphi_{\nu}\, dz^{\mu}\wedge dz^{\nu} \tag{4}$$

where $[(d-\delta)\phi]_{\text{glob}}$ is an expression which remains in (4) if $\sigma=\text{const}$.

In order to compensate additional terms in (4), let us consider the extended operator $\hat{D}$, which acts on the space of the one-forms ϕ_1 (hereafter ϕ_1 denotes an odd form and ϕ_2 an even form):

$$\hat{D}\phi_1 = (d-\delta)\phi_1 + \tilde{\omega}\vee\phi_1 \tag{5}$$

where $\tilde{\omega} = *\frac{1}{2}\omega^{ab}\varepsilon_{ab}$ is the dual Lorentz connection one-form. If h^a_μ is a two-dimensional orthonormal basis, then $\tilde{\omega}_\mu = h^\nu_a(\partial_\nu h^a_\mu - \partial_\mu h^a_\nu)$. Let us note that $\tilde{\omega}$, like the Lorentz connection form ω^{ab}, corresponding to the localizing of the two-dimensional Lorentz group, plays an important role in the structure of the conformal and Lorentz anomalies in two dimensions (Obukhov and Solodukhin, 1990).

For $\tilde{\omega}$ we have the following transformation law under the local transformations (3): $\tilde{\omega}_\mu \to \tilde{\omega}_\mu - \partial_\mu\sigma$. It is interesting to note the following expression for scalar curvature (Obukhov and Solodukhin, 1990):

$$R = 2\nabla_\mu\tilde{\omega}^\mu \tag{6}$$

Thus, we come to the action

$$S = \int * \tfrac{1}{2}(d-\delta)\phi_2 \vee (d-\delta)\phi_2 + \tfrac{1}{2} * \hat{D}\phi_1 \wedge \hat{D}\phi_1 \tag{7}$$

which is the conformal-invariant generalization of the standard string action (Green *et al.*, 1987).

The codifferential operator δ is conjugated to the operator d under the natural scalar product:

$$(\phi, \psi) = \int * \phi\wedge\psi$$

The operator $\hat{D}^+$ conjugated to $\hat{D}$ acts on the space of even forms, and reads

$$\hat{D}^+\phi_2 = -(d-\delta)\phi_2 + \tilde{\omega}\vee\phi_2 \tag{8}$$

where $\vee$ is the Clifford multiplication, defined for the one-forms basis as follows (Kàhler, 1962; Graf, 1978; Becher and Joos, 1983; Benn and Tucker, 1983; Ivanenko and Obukhov, 1985; Ivanenko *et al.*, 1985; Bullinaria, 1986):

$$dz^\mu \vee dz^\nu \equiv dz^\mu \wedge dz^\nu + g^{\mu\nu}$$

The action (7) is invariant also under the local Lorentz rotations of the two-dimensional orthonormal basis:

$$\begin{aligned} h^a_\mu &\to h^a_\mu + \varepsilon^a{}_b h^b_\mu \beta \\ \tilde{\omega}_\mu &\to \tilde{\omega}_\mu + \varepsilon_\mu{}^\nu \partial_\nu \beta \end{aligned} \tag{9}$$

if one determines the action of the Lorentz rotations on the form ϕ_1 as dualization:

$$\varphi_\mu \to \varphi_\mu + \varepsilon_\mu{}^\nu \varphi_\nu \beta \tag{10}$$

where β is the parameter of the transformation.

3. FERMIONIC SECTOR OF THE MODEL

In order to consider the fermionic excitations of the string, one should include the fermionic sector. In the standard string model (Green *et al.*, 1987; Polyakov, 1981) fermions are described by the two-dimensional Weyl spinors. In our model, following to the program of using only differential forms, we will describe fermion fields on the world-sheet by an inhomogeneous differential form:

$$\Psi = \psi + \psi_\mu \, dz^\mu + \tfrac{1}{2} \psi_{\mu\nu} \, dz^\mu \wedge dz^\nu$$

The action is as follows (Kàhler, 1962; Graf, 1978; Becher and Joos, 1983; Benn and Tucker, 1983; Ivanenko and Obukhov, 1985; Ivanenko *et al.*, 1985; Bullinaria, 1986):

$$S = \tfrac{1}{2} \int * \Psi \wedge (d - \delta) \Psi \tag{11}$$

where Ψ components are real and anticommutating.

One can rewrite (11) in the form

$$S = \int d^2 z \sqrt{g} \, (\psi_\mu \partial^\mu \psi + \psi^{\mu\nu} \partial_\mu \psi_\nu) \tag{12}$$

This action is invariant under the global scaling:

$$\begin{aligned} g_{\nu\omega} &\to e^{2\sigma} g_{\mu\nu}, \qquad & \psi_\mu &\to e^{p\sigma} \psi_\mu \\ \psi &\to e^{k\sigma} \psi, \qquad & \psi_{\mu\nu} &\to e^{n\sigma} \psi_{\mu\nu} \end{aligned} \tag{13}$$

where k, p, and n are real numbers, such that

$$k+p=0, \qquad n+p-2=0 \tag{14}$$

Let us assume $\sigma=\sigma(z)$ and $p=1$, $k=-1$, and $n=1$. Then, for the compensation of the $\partial_\mu\sigma$-dependent terms under the variation of (12), one should again consider the operator $\hat{D}$: $\hat{D}\Psi_1=(d-\delta)\Psi_1+\tilde{\omega}\vee\Psi_1$.

In this case the action

$$S=\int *\Psi_2\wedge\hat{D}\Psi_1 \tag{15}$$

is invariant under the local conformal transformations (13) and also under the local Lorentz rotation (19) if we assume that

$$\begin{aligned} \psi &\to \psi+\tfrac{1}{2}\mathscr{E}_{\mu\nu}\psi^{\mu\nu}\cdot\beta \\ \psi_\mu &\to \psi_\mu+\mathscr{E}_\mu{}^\nu\psi_\nu\cdot\beta \\ \psi_{\mu\nu} &\to \psi_{\mu\nu}-\mathscr{E}_{\mu\nu}\psi\cdot\beta \end{aligned} \tag{16}$$

On the other hand, the action (11) is invariant under the local conformal transformations (13) without addition of $\tilde{\omega}$, if we assume $k=0$, $p=0$, and $n=2$ in (13).

4. LOCAL SUPERSYMMETRY

Thus, let us consider the set of the boson forms

$$\phi_2{}^i, \qquad i=1,\ldots,d$$

$$\phi_1{}^A, \qquad A=1,\ldots,N$$

and also the set of the fermion forms

$$\Psi^i, \qquad i=1,\ldots,d$$

$$\tilde{\Psi}^A, \qquad A=1,\ldots,N$$

For these fields the action

$$\begin{aligned} S_0=\int &\tfrac{1}{2}*(d-\delta)\phi_2{}^i\wedge(d-\delta)\phi_2{}^i+\tfrac{1}{2}*\hat{D}\phi_1{}^A\wedge\hat{D}\phi_1{}^A \\ &+*\Psi_1{}^i\wedge(d-\delta)\Psi_2{}^i+*\tilde{\Psi}_2{}^A\wedge\hat{D}\tilde{\Psi}_1{}^A \end{aligned} \tag{17}$$

is invariant under the global supersymmetric transformations

$$\begin{aligned} &\Delta_\alpha \phi_2{}^i = \alpha^{ij}\Psi_2{}^j; \qquad \Delta_\alpha \tilde{\Psi}_2{}^A = -\alpha^{BA}\hat{D}\phi_1{}^B; \qquad \Delta_\alpha \tilde{\Psi}_1{}^A = 0 \\ &\Delta_\alpha \phi_1{}^A = \alpha^{AB}\tilde{\Psi}_1{}^B; \qquad \Delta_\alpha \Psi_1{}^i = -\alpha^{ji}(d-\delta)\phi_2{}^j; \qquad \Delta_\alpha \Psi_2{}^i = 0 \end{aligned} \tag{18}$$

where $(\alpha^{ij}, \alpha^{AB})$ is the set of anticommutating variables.

Localizing these transformations, we find that

$$\Delta_\alpha S_0 = \int * \, d\alpha^{ij} \vee \Psi_2{}^j \wedge (d-\delta)\phi_2{}^i + * \, d\alpha^{AB} \vee \tilde{\Psi}_1{}^B \wedge \hat{D}\phi_1{}^A$$

In order to compensate new terms, it is necessary to introduce the set of anticommutating one-forms (the analogue of the Rarita–Schwinger field) with the transformation law

$$\Delta_\alpha \chi^{ij} = -d\alpha^{ij}, \qquad \Delta_\alpha \chi^{AB} = -d\alpha^{AB} \tag{19}$$

and add to the action the following term:

$$S_1 = \int * \, \chi^{ij} \vee \Psi_2{}^j \wedge (d-\delta)\phi_2{}^i + * \, \chi^{AB} \vee \tilde{\Psi}_1{}^B \wedge \hat{D}\phi_1{}^A$$

Then we have

$$\begin{aligned} \Delta_\alpha(S_0 + S_1) = \int & * \, \chi^{ij} \vee \Psi_2{}^j \wedge \alpha^{ik}(d-\delta)\Psi_2{}^k \\ & + * \, \chi^{AB} \vee \tilde{\Psi}_1{}^B \wedge \alpha^{AC}\hat{D}\tilde{\Psi}_1{}^C + * \, \chi^{ij} \vee \Psi_2{}^j \wedge d\alpha^{ik} \vee \Psi_2{}^k \\ & + * \, \chi^{AB} \vee \tilde{\Psi}_1{}^B \wedge d\alpha^{AC} \vee \tilde{\Psi}_1{}^C \end{aligned} \tag{20}$$

For compensation of the last two terms one should add to the action

$$S_2 = \int \tfrac{1}{2} * \chi^{ij} \vee \Psi_2{}^j \wedge \chi^{ik} \vee \Psi_2{}^k + \tfrac{1}{2} * \chi^{AB} \vee \tilde{\Psi}_1{}^B \wedge \chi^{AC} \vee \tilde{\Psi}_1{}^C$$

and for compensation of the first two terms in (20) one should assume

$$\begin{aligned} \Delta_\alpha \Psi_1{}^i &= -\alpha^{ji}(d-\delta)\phi_2{}^j - \alpha^{ki}\chi^{kj} \vee \Psi_2{}^j \\ \Delta_\alpha \tilde{\Psi}_2{}^A &= -\alpha^{BA}\hat{D}\phi_1{}^B - \alpha^{CA}\chi^{CB} \vee \tilde{\Psi}_1{}^B \end{aligned}$$

Thus, we come to the complete local supersymmetric action:

$$S_{\text{tot}} = S_0 + S_1 + S_2 \tag{21}$$

which is the analogue of the complete standard string action (Brink *et al.*, 1976; Deser and Zumino, 1976). However, contrary to the standard model, the supersymmetry (18), (19) does not touch the gravitational variables (metric).

5. QUANTIZATION AND CONNECTION WITH TOPOLOGICAL QUANTUM FIELD THEORY

The quantization of our model is defined by the functional integral

$$Z=\int [Dh^a_\mu][D\chi][D\Psi][D\tilde{\Psi}][D\phi_1][D\phi_2] \times \exp(-S_{\rm tot})$$

Let us start with the calculation of the integral

$$Z'=\int [D\Psi][D\tilde{\Psi}][D\phi_1][D\phi_2]\exp(-S_{\rm tot})$$

which is the partition function for the action (21) if we consider the gravitational (h^a_μ) and supergauge (χ) fields as external.

Since $S_{\rm tot}$ is quadratic over fields, the calculation of the functional integral gives the expression for Z' in terms of superdeterminants (Berezin, 1979):

$$Z'=(S\det R\cdot S\det \tilde{R})^{-1/2} \tag{22}$$

where the operator R ($\tilde{R}$) has the following structure:

$$R=\begin{pmatrix} R_1 & R_2\\ R_3 & R_4\end{pmatrix},\qquad \tilde{R}=\begin{pmatrix} \tilde{R}_1 & \tilde{R}_2\\ \tilde{R}_3 & \tilde{R}_4\end{pmatrix}$$

with

$$R_1=(-(d-\delta)^2_{(2)}\delta^{ij});\qquad R_2=(0\quad -(d-\delta)_{(1)}\chi^{ij})$$

$$R_3=\begin{pmatrix} 0\\ -\chi^{ji}\vee(d-\delta)_{(2)}\end{pmatrix};\qquad R_4=\begin{pmatrix} 0 & (d-\delta)_{(2)}\delta^{ij}\\ (d-\delta)_{(1)}\delta^{ij} & -\chi^{ki}\vee\chi^{kj}\end{pmatrix}$$

$$\tilde{R}_1=(\hat{D}^+\hat{D}\delta^{AB});\qquad \tilde{R}_2=(0\quad \hat{D}^+\chi^{AB})$$

$$\tilde{R}_3=\begin{pmatrix} 0\\ -\chi^{BA}\vee\hat{D}\end{pmatrix};\qquad \tilde{R}_4=\begin{pmatrix} 0 & \hat{D}\delta^{AB}\\ -\hat{D}^+\delta^{ab} & -\chi^{CA}\vee\chi^{CB}\end{pmatrix}$$

The superdeterminant is expressed as (Berezin, 1979)

$$S\det R=\det R_1\det{}^{-1}(R_4-R_3R_1^{-1}R_2)$$

Hence,

$$S\det R=[\det\Delta_0\det\Delta_2\det{}^{-1}\Delta_1]^{d/2} \tag{23}$$

$$S\det\tilde{R}=[\det\hat{D}^+\hat{D}\cdot\det{}^{-1}\hat{D}\hat{D}^+]^{N/2} \tag{24}$$

where $\Delta_k=-(d-\delta)^2_{(k)}$ is the Beltrami–Laplace operator on the k-forms.

Thus, we have for Z'

$$Z' = [T(M)]^{d/2}[\tilde{T}(M)]^{-N/2} \tag{25}$$

where

$$T(M) = [\det^{-1} \Delta_1 \det \Delta_0 \det \Delta_2]^{-1/2} \tag{26}$$

$$\tilde{T}(M) = [\det^{-1} \hat{D}^+ \hat{D} \det \hat{D}\hat{D}^+]^{-1/2} \tag{27}$$

It is interesting to note that Z' is independent of the supergauge field χ and is expressed in terms of $T(M)$, the topological invariant of the manifold M (the string world-sheet)—the Ray–Singer torsion (Ray and Singer, 1971; Schwarz, 1978). For the two-dimensional manifold, $T(M) = 1$, since in this case $\det \Delta_1 = \det \Delta_0 \det \Delta_2$. Similarly, we have $\det \hat{D}^+ \hat{D} = \det \hat{D}\hat{D}^+$ and consequently $T(M) = 1$. It should be noted that the mutual cancellation of the boson and fermion determinants is a typical property of the so-called topological quantum field theory (Witten, 1988).

Thus, we find that Z', a functional of χ and h^a_μ, turns out to be a topological invariant, i.e., it does not change under the local variations of these fields. Consequently, the suggested model is an example of the topological quantum field theory of Witten (1988). [It is interesting to note that in Witten (1988) the action (11) is also used for the description of the fermionic sector.]

6. CANCELLATION OF CONFORMAL ANOMALIES

Let us return to the calculation of the functional integral Z, and choose the conformal-Lorentz gauge, in which

$$h^a_\mu = e^\sigma(\delta^a_\mu \cos \alpha + \mathscr{E}^a{}_\mu \sin \alpha)$$

$$d * \chi = 0$$

where $\chi \equiv (\chi^{ij}, \chi^{AB})$. The latter equation means that $\chi = * d\beta$, where $\beta \equiv (\beta^{ij}, \beta^{AB})$.

For the functional measure over h^a_μ we have the standard expression (Polyakov, 1981; Nazarowski and Obukhov, 1987)

$$[Dh^a_\mu] = [D\alpha][D\sigma][D\xi^\mu] \det{}^{1/2} \hat{L} \tag{28}$$

where ξ^μ is a vector field, the generator of the diffeomorphism group,

$$L^\mu_\nu \xi^\nu = (-\nabla_\alpha \nabla^\alpha \delta^\mu_\nu + [\nabla^\mu, \nabla_\nu])\xi^\nu$$

Note that $\int [D\xi^\mu]$ gives the volume of the diffeomorphism group.

For the integration measure over χ we obtain

$$[D\chi]=[D\beta](\det\Delta_0)^{-(P_1+P_2)}$$

where P_1 is the number of $\{\alpha^{ij}\}$ and P_2 is number of $\{\alpha^{AB}\}$.

Thus, we have

$$Z=\int[D\alpha][D\beta][D\sigma]\det{}^{1/2}\hat{L}\,(\det\Delta_0)^{-(P_1+P_2)}Z' \tag{29}$$

where Z' is independent of σ, β, and α.

The determinants are well known,

$$\ln\det\Delta_0=\frac{1}{6\pi}I_0[\sigma]$$

$$\ln\det\hat{L}=\frac{13}{3\pi}I_0[\sigma]$$

where $I_0[\sigma]=-\frac{1}{2}\int d^2z\,\partial_\mu\sigma\,\partial^\mu\sigma$.

The Lorentz and supergauge anomalies, as is seen from (29), are absent, so the integrals over $[D\alpha]$ and $[D\beta]$ give the volumes of the corresponding groups.

Hence, we get

$$Z=\int[D\sigma]\exp\frac{C}{6\pi}I_0[\sigma]$$

where $C=13-(P_1+P_2)$.

For the cancellation of conformal anomalies it is necessary to have $C=0$ or $P_1+P_2=13$. Up to this point $\{\alpha^{ij}\}$ and $\{\alpha^{AB}\}$ were arbitrary matrices. Let us assume now that

$$\alpha^{ij}=\begin{cases}\alpha, & i=j\\ \alpha^{ij}=-\alpha^{ji}, & i\neq j;\quad i=1,\ldots,4\end{cases}$$

$$\alpha^{AB}=\begin{cases}\alpha, & A=B\\ \alpha^{AB}=-\alpha^{BA}, & A\neq B;\quad A=1,\ldots,4\end{cases}$$

In this case the infinitesimal supersymmetry parameters take values in the algebra of the group $O(4)\otimes O(4)\otimes O(1)$ and consequently $P_1+P_2=13$. Thus, we finally have that $N=d=4$.

In our model the zero-forms φ^i realize the embedding of the string world-sheet into d-dimensional space-time, so the model suggested here is consistent (the anomalies are absent) directly in four-dimensional space-time.

7. CONCLUSION

We have considered the string model, using differential forms on the string world-sheet. This model allows for local supersymmetry, which, like the supersymmetry in Witten (1988), leads to "topological quantum field theory."

Scalar components of the boson form ϕ^i realize the embedding of strings into d-dimensional space-time, and have a direct geometric sense. It was shown that the conformal anomalies are absent if $d=4$.

We will not discuss the geometric interpretation of the other components of the boson form ϕ^i. Evidently one can consider them simply as the terms of the supermultiplet.

The canonical quantization and the no-ghost theorem will be considered elsewhere.

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